Practice session: Cosmology

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Don't forget to give motivations and/or derivations for your answers!

This practice exam is longer than the actual exam will be. Try to focus on topics you find difficult. Some topics might be above the difficulty of the course. If you have any questions regarding the exam or related to cosmology do not hesitate to ask them.

1. Cosmic dynamics can be described by three equations: the Friedmann equation, the acceleration equation and the fluid equation, which are respectively given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}(\epsilon_m + \epsilon_r) - \frac{kc^2}{R_0^2 a^2} + \frac{\Lambda}{3}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P) + \frac{\Lambda}{3}$$
$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0,$$

where $P = \omega\epsilon$ is the equation of state between pressure P and energy density ϵ.

(a) Use the fluid equation for a flat universe with no cosmological constant to show that the energy density evolves with the expansion factor as

$$\epsilon = \epsilon_0 a^{-3(1+\omega)},$$

with $\omega \neq 1$.

(b) Use the Friedmann equation and the answer in (c) to infer that, for the same type of universe, the expansion factor evolves in time as

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+\omega)}}.$$

(Hint: $\epsilon_0 = \frac{3c^2 H_0^2}{8\pi G}$.)

(c) Use this to show that the Hubble parameter evolves as

$$H_0 = \left(\frac{\dot{a}}{a}\right)_{t=t_0} = \frac{2}{3(1+\omega)} t_0^{-1}.$$

What is the physical meaning of t_0?

(d) Calculate the energy density evolution for a matter dominated universe ($\omega = 0$), and for a radiation dominated universe ($\omega = \frac{1}{3}$). For both universes, physically explain the dependence on a.

2. Explain the following cosmological concepts:

(a) Cosmological principle

(b) Inflation

(c) Copernican principle
(d) Weak/Strong Anthropic principle
(e) Metric

3. Distances in the Universe.
There are several types of distance measures available, a few of these are the so called proper distance (d_p), luminosity distance (d_L) and the angular-diameter distance (d_A).
   (a) What is the difference between the proper distance d_p and the co-moving distance r found in the RW-metric, which is the physical distance?
   (b) Briefly explain the difference between these three distance measures.
   (c) What is the effect of the curvature of space time on the luminosity distance?
   (d) How does the expansion of space affect the luminosity distance?
   (e) Given these considerations, write down an expression for the luminosity distance.
   (f) Do the same steps for the angular-diameter distance; what is the relation between d_A and d_L?
   (g) For a Λ cosmology the angular-diameter distance reaches a maximum after which objects start growing in size with increasing redshift, explain this phenomenon.

4. Big Bang Problems
   (a) There are several fundamental problems in the standard Big Bang model, state three of them.
   (b) Give a possible solution for at least two of the problems you've listed.

5. Event Horizon
   (a) What is the difference between an event horizon and a particle horizon?
   (b) Bulbasaur, Squirtle and Charmander decide to use their cosmological knowledge to try and send information from beyond others event horizon. Bulbasaur and Squirtle are not within each others horizons, Charmander however, is located in both. Therefore, Bulbasaur had the clever idea to send a signal to Charmander, who would then send it to Squirtle. Explain to Bulbasaur why this will not work and why he is thus clearly the worst starter.
   (c) Compute the particle horizon at a given time t for the cosmologies given in Table 1
   (d) Three of these cosmologies are also known under certain names. Which three are these and what are their names.

	Ω_Λ	Ω_M	Ω_R
Cosmology 1	0	0	0
Cosmology 2	1	0	0
Cosmology 3	0	1	0
Cosmology 4	0	0	1

Table 1: Different Cosmologies

6. Nucleosynthesis
   (a) What is the Deuterium bottleneck?
   (b) Explain the significance of the ~ 1 MeV energy scale in the early universe.
   (c) Explain how neutrons survive the hot big bang, even though free neutrons decay in 15 minutes.

7. Structure formation in the Universe.
   (a) For a cold dark matter dominated universe, use the perturbed fluid equations (Equation 4) to obtain the second order partial differential equation for the density perturbation δ given by
   $$\frac{\partial^2\delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial\delta}{\partial t} = \frac{3}{2}\Omega_0 H_0^2 \frac{1}{a^3}\delta$$
   (Hint: Consider only linear terms)
   (b) Now consider this universe to be an Einstein-deSitter universe, solve the differential equation by using the ansatz $\delta(t) \propto t^\alpha$.
   (c) Do the same steps again but this time for an empty universe. What does this tell you about the growth of structure in an open universe?

$$\frac{\partial\delta_i}{\partial t} + \frac{(1+w_i)}{a}\vec{\nabla}\cdot(1+\delta_i)\vec{v} = 0 \tag{4a}$$

$$\frac{\partial\vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} = -\frac{\nabla P_i}{(1+w_i)\rho_i} - \frac{1}{a}\vec{\nabla}\phi \tag{4b}$$

$$\nabla^2\phi = 4\pi G a^2 \rho_i \delta_i \tag{4c}$$